Theoretical Parameter Study of Aerodynamic Vectoring Particle Sorting

Dane N. Jackson
Barton L. Smith

Mechanical and Aerospace Engineering, Utah State University, Logan, UT 84322

1 Introduction

Particle manufacturing processes are often incapable of making uniform-sized particles. If a uniform size is necessary, sorting of the particles is required. In addition, sorting is often used as a diagnostic tool to determine the number of particles of a given size in a sample. When detection of particles such as pathogens is required, it is beneficial to concentrate a sample, which can be achieved using a sorting apparatus.

While methods for particle sorting may differ greatly from one application to another, the principles involved are often similar. For over a century, particle sorting has been accomplished through use of plate, and more recently, virtual impactors. The technique relies on turning a free jet sharply without extended control surfaces. The flow turning results in a balance of particle inertia and several forces (pressure, drag, added mass, and body forces) that depend on particle size and density. The present paper describes a theoretical study of particle sorting in a turning flow. The purpose of this study is to extend AVPS to parameter spaces other than those that are currently under investigation. Spherical particles are introduced into a turning flow in which the velocity magnitude increases like r. The trajectory of each particle is calculated using the particle equation of motion with drag laws that are appropriate for various Knudsen number regimes. Large data sets can be collected rapidly for various particle sizes, densities, turning radii, flow speeds, and fluid properties. Ranges of particle sizes that can be sorted are determined by finding an upper bound (where particles move in a straight line) and a lower bound (where particles follow flow streamlines). It is found that the size range of particles that can be sorted is larger for smaller turning radii, and that the range moves toward smaller particles as the flow speed and the particle-to-fluid density ratio are increased. Since this flow is laminar and 2-D, and particle loading effects are ignored, the results represent a “best case” scenario.

2 AVPS Model

Predicting a particle’s trajectory and sorting a particle by size are made possible by a calculation of the particle forces found in the governing equations of particle motion. Models of the forces on a particle in motion date back a century. Millikan [6–8] performed a famous oil drop experiment in the early twentieth century to provide empirical data that was used to modify the Stokes drag force equation:
where the constants derived from Epstein’s coefficients calculated by

\[ \text{Drag} = -6\pi\mu a [V(t) - u_i(Y(t), t)] \]  

(1)

where \( a \) is the particle diameter, \( u_i \) is the fluid velocity at the particle location \( Y(t) \), \( V \) is the particle velocity, and \( \mu \) is the dynamic viscosity.

The particle size relative to the mean free path of the fluid is accounted for in the more sophisticated model [10]

\[ \text{Drag} = \frac{-6\pi\mu a [V(t) - u_i(Y(t), t)]}{1 + Kn(a + QeKn)} \]  

(2)

where \( A, B, \) and \( Q \) are Cunningham slip coefficients, and \( Kn \) is the Knudsen number. \( Kn = 2\lambda / a \), where \( \lambda \) is the mean free path. The equation is valid for Knudsen numbers ranging from 0.01 to 100. Knudsen numbers greater than 100 are described as "large" while values less than 0.01 are described as "small." Other improvements came from Basset [11] for Knudsen numbers much less than unity and by Epstein [12] in 1924 for Knudsen numbers much greater than unity.

For the present study, depending on particle size, one of three analytical models is used to calculate the particle drag term in the particle equation of motion. Particles of Knudsen numbers smaller than 0.01 have a drag calculated by the following equation derived by Basset [11]:

\[ D = \frac{-3\pi\mu a [V(t) - u_i(Y(t), t)]}{m_p + \frac{m_f}{2}} \left[ 1 - \frac{Kn}{\sigma} (2 - \sigma) \right] \]  

(3)

where \( \sigma \) is the accommodation coefficient. Equation (3) is used to calculate acceleration due to drag by dividing the force by the particle mass and half the displaced fluid mass. Particles of Knudsen numbers between 0.01 and 100 have a drag calculated by

\[ D = \frac{-3\pi\mu a [V(t) - u_i(Y(t), t)]}{m_p + \frac{m_f}{2}} \left[ 1 + Kn (\alpha + QeKn) \right] \]  

(4)

where the constants \( \alpha, Q, \) and \( \beta \) are the Cunningham slip coefficients [6–8] for water particles in air.

The drag of particles of Knudsen numbers greater than 100 is calculated by

\[ D = \frac{-3\pi\mu a [V(t) - u_i(Y(t), t)] (8 + \pi \sigma)}{18Kn m_p + \frac{m_f}{2}} \]  

(5)

derived from Epstein’s [12] theoretical solution.

In 1956, Corrsin and Lumley [13] provided a more concise set of equations of motion that included other forces such as pressure effects (the force on a sphere due to the pressure field of the undisturbed flow) and added mass. In the present paper, the forces acting on a particle under acceleration will be calculated using the particle equation of motion developed by Maxey and Riley [14]:

\[ \frac{dV(t)}{dt} = P + M + D + B \]  

(6)

\[ P = m_f \left( \frac{Du}{Dt} - \nu \nabla^2 u_i \right) \]  

(7)

\[ M = -\frac{m_f}{2} \frac{d}{dt} \left[ V(t) - u_i(Y(t), t) \right] \]  

(8)

\[ B = (m_p - m_f)g \]  

(9)

These forces are referred to as the pressure (Eq. (7)), added mass (Eq. (8)), drag (Eqs. (3)–(5)), and body forces (Eq. (9)). The pressure term represents the force exerted by the pressure gradient at the location of the particle if the particle had no influence on the flow. The added mass term represents the force required to accelerate a mass of fluid from the ambient at the particle location. Drag is the force due to differences in particle and fluid velocities. The body force is the force due to gravity. These forces are illustrated in Fig. 2. The particle equation of motion is solved using a fourth-order Runge-Kutta method. The horizontal and vertical velocities of the particle are compared to the velocity of the flow for each point to calculate the forces.

Our flow model is a two-dimensional solid body rotation meant to simulate the turning region found by Bettridge et al. [4], and, more directly, the recent result of Humes et al. [5] shown in Fig. 1(b). We wish to learn the effects of many fluid, geometric, and particle parameters. Thousands of test cases are required to examine the effects of each parameter on the others. A simplified flow field allows for the study of a large number of cases in a timely manner.

The magnitude of the velocity increases linearly with \( r \). As each streamline maintains a constant angular velocity independent of turning radius, shear effects are not present. Particles are introduced at the start of the turning and initially have the same velocity as the fluid.

In order to determine the particle size and density limits for AVPS, rejected particle path cases are categorized as follows: “Streamline” cases are those in which, either due to its small diameter or small density, a particle’s deviation from a fluid streamline is less than a specified amount. An example of a streamline case is the innermost path shown in Fig. 3. “Straight line” cases are those in which a particle’s inertia carries it through the domain with little or no deviation from its initial path. A straight line example is the straightest path seen in Fig. 3. Both streamline and straight line cases represent particles that cannot be collected into sorted groups, and will therefore constitute the sorting limits.
3 Results

Six cases of particles in air were modeled to find the range of sortable particles for particle-to-fluid density ratios of 100:1, 500:1, 1000:1, 2500:1, 5000:1, and 10,000:1. Each case included several subcases that tracked a single particle through the flow. The inlet velocity and the inlet distance from the flow center, or turning radius \( r \), were varied. The inlet velocities for each case were \( U = 1, 5, \) and 10 through 100 m/s in increments of 10 m/s. The distances to the flow center were 2 mm to 9 mm in increments of 1 mm.

The paths of acceptable particle diameters are shown in Fig. 4 for particle sizes from three different cases. The \( x \)-axis is the horizontal distance and the \( y \)-axis is the vertical. The particle enters the flow in the lower-left portion of the field. It can be seen that the particle’s deviation from the streamlines increases with particle diameter.

The effect of inlet velocity is shown in Fig. 5. The paths of particles with the same diameter, density ratio, and turning radius are shown for various inlet velocities. Particles deviate more from the streamlines with increasing inlet velocity for a given turning radius.

The effect of particle-to-fluid density ratio is shown in Fig. 6. Paths for particles of the same diameter, inlet velocity, and turning radius are shown for various particle-to-fluid density ratios. Particles deviate more from the streamlines with increasing density ratio for a given turning radius. This is expected since particles of higher particle-to-fluid density ratios will have relatively more inertia.

The sorting limits for the cases of particle-to-density ratios of 100:1 and 10,000:1 are shown in Figs. 7 and 8, respectively. The lines plotted with solid symbols represent the upper (straight line) limits and open symbols for the lower (streamline) limits for various inlet speeds. Figures 7 and 8 each demonstrate that with increasing turning radius, the streamline limit increases while the straight line limit decreases. A wider range of particles can be sorted as turning radius is decreased. This is because the sharper turn generates larger accelerations that are able to accelerate larger particles more while requiring less inertia for smaller particles to remain entrained. The cases not shown follow the same trends.
It can also be seen that the streamline and straight line limits decrease together as inlet velocity is increased. Figure 9 illustrates the streamline and straight line limits of different particle-to-fluid density ratios for inlet velocities of 50 m/s. Generally, the range of sizes that can be sorted remains constant between one and two orders of magnitude depending on the turning radius. Larger density ratios shift both the lower and upper limits downward.

It should be noted that the results presented in this paper apply only to an ideal case where the flow is laminar and 2-D, the particles are spherical, and particle loading is small. Departure from any of these conditions will likely result in similar mean results, but may cause large standard deviations in the particle size at any location.

The upper and lower limits of particle sorting have been found to be a function of the turning radius, the particle density, and the inlet velocity. Based on dimensional analysis and the data from this study, the two limits of the sorting range are cast dimensionlessly. Specifically, the upper streamline limit relative to the turning radius is found to be

$$\frac{a}{r}_{\text{streamline}} = \left( \frac{1.38 \rho}{\text{Re} \rho_p} \right)^{0.5}$$

where $\text{Re} = \frac{Ur}{\nu}$, $U$ is the inlet velocity, $\nu$ is the fluid kinematic viscosity, $r$ is the initial distance to the flow center, and $\rho_p/\rho$ is the particle-to-fluid density ratio. The fit of the data from the present study to Eq. (10) is shown in Fig. 10. The equation adequately represents all of the data.

The straight line limit diameter is normalized by the mean free path and is a function of the same parameters (see Fig. 11)

$$\text{Kn} = \left[ \frac{\rho_p}{\rho} \frac{\text{Re}}{\text{Re}^{0.5}} \frac{1.8}{r} \frac{\text{Re}^{0.5}}{\lambda} \right]^{0.5}$$

where $\text{Re}_k = U \lambda / \nu$, $\gamma = 3.3 \times 10^{-10}$, and $\lambda$ is the mean free path of air.
articles of various sizes could have inside the window. The symbols in Fig. 12 represent all possible trajectory angles that the par-

“data” are also collected in the same-sized region. The gray band acquired data over a small but finite region

force for a short time, and therefore the trajectory angle of each

this location, the particles in both the experiment and the theoret-

A VPS apparatus. Since the experimental flow is not infinite in

100 m/s would require a turning radius of 0.2 mm and a jet less

These trends are quantified in a pair of equations fit to the

5 Conclusions

A theoretical model has been constructed to determine the lim-

its of particle sorting by rapidly turning a free jet. The theoretical

model is verified by comparing the trajectory of a wide range of

particle sizes to experimental results. It has been demonstrated

that sorting particles over a wide range of diameters is possible

using the AVPS arrangement. Particles that are too large to be

sorted move in a straight line in a turning flow. Particles that are

too small follow flow streamlines. The streamline and straight line

limits decrease as the inlet velocity is increased. The streamline

and straight line limits decrease as the particle-to-fluid density

ratio is increased. A wider range of particles can be sorted as the

 turning radius is decreased. This effect is especially important for

small turning radii.

These trends are quantified in a pair of equations fit to the

theoretical results—one for the lower limit (Eq. (10)), and a sec-

ond for the upper limit (Eq. (11)). These equations are useful for

scaling a sorting apparatus. For instance, if one wishes to sort

particles five times the density of water down to 0.5 μm, a flow

speed of 30 m/s and a turning radius of 2 mm would suffice. A jet

with an exit velocity of 30 m/s would need to be no larger than

2.5 mm in order to remain laminar. On the other hand, if it were

desired to sort to 0.1 μm of the same material, a flow speed of

100 m/s would require a turning radius of 0.2 mm and a jet less

than 0.75 mm wide. Clearly, the practicality of this method de-

creases rapidly with particle size in the neighborhood of 1 μm.

It should be noted that the results presented in this paper apply

only to an ideal case where the flow is laminar and 2-D, the

particles are spherical, and particle loading is small. Departure

from any of these conditions will likely result in similar mean

results, but may cause large standard deviations in the particle size

at any location.

References


Fig. 11 Comparison to model results for Knudsen number (Eq. (11))

Fig. 12 Experimentally measured particle trajectory angles (glass spheres, ρp/ρ=600) compared to the present results indicated by the shaded regions


